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# Engineering Notes

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## Dynamics of a Flexible Space Tether Equipped with a Crawler Mass

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### I. Introduction

THE dynamic equations of a flexible space tether system equipped with a crawler mass are developed here and the behavior of the crawler mass is studied by numerical simulation. Tethers represent a new space technology that can be applied to build space structures. The concept of tether systems goes back to 1857 when Tsiolkovsky in Russia conceived of tethers as a means to generate artificial gravity [1]. Over a century later, tethers receive attention in planning future space systems due to their advantageous features such as 1) there is a potential for large reductions in the weight of very large space structures by employing tethers as structural elements; 2) very long tethers can be packed in a small compact space during launch, 3) it is possible to deploy tethers to a length of more than 10,000 km by taking advantage of the strength provided by new materials such as Kevlar or Dyneema, and 4) tether structures can be constructed automatically without requiring much assistance from astronauts, simply deploying the tethers from the spools on which they are compactly wound. For these reasons various future missions employing tether technology have been proposed [2,3].

Space systems in Earth orbit operate in a space environment that exposes them to micrometeoroid and orbital debris, atomic oxygen, and radiation. A long tether even with a small cross-sectional dimension has a large area exposed to collision with debris. Hence, a drawback of using tethers is the possibility that they may be severed by such a collision. Two approaches to address this issue are 1) take a passive approach that provides tether redundancy by using the "hoytether" [4] or tape tether [5], and 2) take an active approach employing a crawler to inspect and maintain the tether system. The present work is devoted to studying the dynamics of a crawler moving on a flexible tether system. The dynamics of a climber on space elevator system [6] could be an extension of the present analysis, although the present study does not consider an active driving force for the crawler motion.

The purpose of the present study is to analyze the dynamics of the flexible tether system equipped with a crawler mass and thus to construct a numerical simulator to study the dynamic behavior. The following assumptions are made: 1) only planar motion is treated for the space tether system; 2) only the gravitational force of the center of attraction is considered, and such effects as atmospheric drag and solar pressure are neglected; 3) the distributed tether mass is modeled as many particles connected by massless strings, and one additional particle to represent the crawler mass; 4) the length of a tether element is constant; and 5) a single crawler mass moves on the tether but has no driving force.

Previous research analyzing tether systems equipped with a crawler has neglected tether flexibility in the analysis and thus the movement of the crawler system is not simulated completely. Typically one of two models is used in the formulations due to the difficulty of analysis: one model assumes a rigid tether and the crawler mass moving along it [7,8], and the other assumes that the crawler mass is positioned at a connecting point of two rigid tethers [9]. These models, however, do not reflect the full dynamic features of a flexible tether system with a crawler mass moving on it. The present study formulates the dynamics including flexibility modeled using many mass elements connected by strings. The problem is made more difficult by the fact that the crawler must be constrained to move on the tether whose ends are floating in space.

### II. Equations of Motion

The equations of motion are formulated using Kane's method together with methods from mechanics to include the constraints [10]. The formulation of the present problem assumes that the crawler mass moves along the tether purely in response to such conservative forces as gravitational and inertial forces. Nonconservative forces such as a driving force or frictional forces are not considered because their inclusion involves a substantial increase in the complexity of the analysis.

The flexible tether is modeled as a set of many particles with massless strings connecting them. Dynamic modeling of the motion of the crawler mass along such a tether model requires handling two different situations: 1) when the crawler mass moves on the string part of the tether model, and 2) when the crawler mass moves through the particle part of the tether model. The crawler mass will pass through a particle position at a joint point of two strings in a natural and continuous manner satisfying physical conditions of motion, if no nonconservative forces are applied. Conservation laws assist the modeling as the crawler reaches a joint point, moves over the joint, and departs the joint to the next string. The initial condition to move onto the next string element is determined to conserve both the energy and momentum of the motion at the time periods before and after the transit of the crawler mass through the joints.

A simulator of the tether satellite system equipped with the crawler mass is programmed based on the preceding formulation and the dynamic behavior is analyzed numerically. Figure 1 shows the model used for the orbiting flexible tether with a crawler mass, consisting of $N$ particles with mass $m_i$ ($i = 0, 1, \ldots, N-1$) and massless strings of length $\ell_i$ ($i = 1, 2, \ldots, N-1$) connecting the particles. The rotation of string $i$ with length $\ell_i$ is denoted by $\theta_i$ ($i = 1, 2, \ldots, N-1$) measured with respect to the orbit radius vector $\mathbf{R}_c$. The crawler mass is assumed to be a particle with mass $m_N$ and its position is described by the vector $\mathbf{l}_N$ with origin at the $(N-1)$st end body as is shown in Fig. 1. The crawler mass must be placed on the $k$th string and the position of the crawler is also denoted by a point with distance

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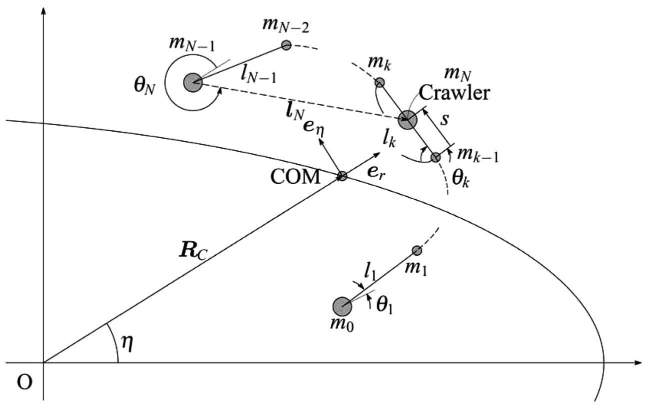


**Fig. 1 The model of a tether satellite system equipped with a crawler mass.**

$s$ from the particle $m_{k-1}$ and on the $k$th string element connecting the particles $m_{k-1}$ and $m_k$. These two descriptions of the crawler mass position must be made consistent.

The equations of motion for the system shown in Fig. 1 are formulated using Kane's method for all $N$ masses and strings. Then the following two constraints are incorporated in the formulation by using the method of constrains [10]: 1) the lengths of tether string element $\ell_i$ $(i = 1, 2, \ldots, N-1)$ are constant; and 2) the crawler mass $m_N$ is on the tether element $\ell_k$ connecting the particles $m_{k-1}$ and $m_k$.

The equations of motion for the model of the tether consisting of particles connected by strings are obtained as follows:

$$\ddot{R}_c = R_c \dot{\eta}^2 + Q_R \tag{1}$$

$$\ddot{\eta} = \frac{1}{R_c}(-2\dot{R}_c \dot{\eta} + Q_\eta) \tag{2}$$

$$\ddot{\ell}_i = \ell_i(\dot{\theta}_i + \dot{\eta})^2 + Q_{li}\cos\theta_i + Q_{\theta i}\sin\theta_i \qquad (i = 1, \ldots, N) \tag{3}$$

$$\ell_i(\ddot{\theta}_i + \ddot{\eta}) = -2\dot{\ell}_i(\dot{\theta} + \dot{\eta}) - Q_{li}\sin\theta_i + Q_{\theta i}\cos\theta_i \qquad (i = 1, \ldots, N) \tag{4}$$

where $R_c$ and $\eta$ are the radius and the true anomaly of the orbit, $\ell_i$ and $\theta_i$ are the length and rotation of the $i$th element of tether strings, $\ell_N$ and $\theta_N$ denote the length and the rotation of the vector $\mathbf{l}_N$, and $Q_{\ell i}$ and $Q_{\theta i}$ denote the generalized forces applied along the length and rotation of the string element, respectively, and the overdot denotes differentiation with respect to time. Equations (1) and (2) describe the motion of the center of mass of the system in the orbital plane and Eqs. (3) and (4) the motion of tether particles and the crawler mass.

Let us now introduce the constraints that the lengths of the tether strings are all equal and of length $(L_c)$

$$\Phi_i := \ell_i - L_c = 0 \qquad (i = 1, \ldots, N-1) \tag{5}$$

Also, in order that the crawler mass stay on the tether, the position of the crawler mass must coincide for both descriptions that the mass is on the $(k-1)$st string and that given by the vector $\mathbf{l}_N$ connecting the $(N-1)$st end body and the crawler mass on the $k$th string. The constraint that the crawler mass is placed on the $k$th string is

$$\Phi_N := \sum_{i=k+1}^{N-1} \ell_i \sin(\theta_i - \theta_k) + \ell_N \sin(\theta_N - \theta_k) = 0 \tag{6}$$

The distance $s$ of the crawler mass from the position of the $(k-1)$ st particle measured along the $k$th string is described by using the vector $\mathbf{l}_N$ as

$$s := \ell_k + \sum_{i=k+1}^{N-1} \ell_i \cos(\theta_i - \theta_k) + \ell_N \sin(\theta_k - \theta_N) \tag{7}$$

where $\ell_N := |\mathbf{l}_N|$. The description for the velocity and the acceleration of a crawler mass along the $k$th string, that is, the time derivatives of the length $s$, are then obtained from Eq. (8) when the dynamic parameters are determined.

The Lagrange function $L$ is obtained including the constraints $\boldsymbol{\Phi} = [\Phi_1 \cdots \Phi_N]^T$ as

$$L = (1/2)\dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}} - U + \boldsymbol{\lambda}^T \boldsymbol{\Phi} \tag{8}$$

where $\mathbf{q} = [R_c, \eta, \ell_1, \ldots, \ell_N, \theta_1, \ldots, \theta_N]^T$ is the generalized coordinates, $\mathbf{M}$ is the mass matrix, $U$ is the potential function, $\boldsymbol{\Phi} = [\Phi_1 \cdots \Phi_N]^T$ are the constraints, and $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_N]^T$ is the Lagrange multiplier. The equations of motion from Lagrange equations are

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\lambda}^T \boldsymbol{\Phi} = \mathbf{Q} \tag{9}$$

where $\mathbf{Q}$ denotes the generalized force. Differentiating the constraints $\boldsymbol{\Phi} = [\Phi_1 \cdots \Phi_N]^T$ twice with respect to time, we obtain

$$\boldsymbol{\Phi}_q \ddot{\mathbf{q}} + (\boldsymbol{\Phi}_q \dot{\mathbf{q}})_q + 2\boldsymbol{\Phi}_{qt}\dot{\mathbf{q}} + \boldsymbol{\Phi}_{tt} = 0 \tag{10}$$

where the subscripts $q$ and $t$ denote partial differentiation with respect to the corresponding variables. Equations (9) and (10) can be combined into a single matrix equation as

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_q^T \\ \boldsymbol{\Phi}_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ \boldsymbol{\gamma} \end{bmatrix} \tag{11}$$

where $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_N]^T := -\lfloor (\boldsymbol{\Phi}_q \dot{\mathbf{q}})_q + 2\boldsymbol{\Phi}_{qt}\dot{\mathbf{q}} + \boldsymbol{\Phi}_{tt} \rfloor$. Using Lagrange multipliers to coordinate the constraints (5) and (6) with Eqs. (1–4), the equations of motion for the present system are finally obtained as follows [10]:

$$\ddot{\mathbf{q}} = \left\{ \mathbf{I} - \mathbf{M}^{-1}\boldsymbol{\Phi}_q^T \left( \boldsymbol{\Phi}_q \mathbf{M}^{-1} \boldsymbol{\Phi}_q^T \right)^{-1} \boldsymbol{\Phi}_q \right\} + \mathbf{M}^{-1}\boldsymbol{\Phi}_q^T \left( \boldsymbol{\Phi}_q \mathbf{M}^{-1} \boldsymbol{\Phi}_q^T \right)^{-1} \boldsymbol{\gamma} \tag{12}$$

$$\boldsymbol{\lambda} = \left( \boldsymbol{\Phi}_q M^{-1} \boldsymbol{\Phi}_q^T \right)^{-1} \left( \boldsymbol{\Phi}_q \mathbf{M}^{-1} \mathbf{Q} - \boldsymbol{\gamma} \right) \tag{13}$$

Let us now analyze the motion of the crawler crossing any joint point, that is, a particle connecting two adjoining strings. The forces applied on the crawler mass are assumed to consist of conservative force only, including gravitational and inertial forces, producing a natural and continuous motion of the crawler mass over any particle. The initial condition after the crawler mass has crossed over a particle is determined to conserve energy and momentum of the crawler mass from before to after transiting the particle. The following equations are obtained relating the state variables of the crawler mass before and after the particle crossing:

$$\dot{\theta}_k^+ = \dot{\theta}_k^- - \frac{m_N\{(m_0 + m_{N-1})(m_N + m_k) + 2m_0 m_{N-1}\}}{l_k m_0 (m_N + m_k)(m_{N-1} + m_N + m_k)} \dot{s} \tan\left( \frac{\theta_{k+1} - \theta_k}{2} \right) \tag{14}$$

$$\dot{\theta}_{k+1}^+ = \dot{\theta}_{k+1}^- - \frac{m_N\{(m_0 + m_{N-1})(m_N + m_k) + 2m_0 m_{N-1}\}}{l_{k+1} m_{k+1} (m_N + m_k)(m_0 + m_N + m_k)} \dot{s} \tan\left( \frac{\theta_{k+1} - \theta_k}{2} \right) \tag{15}$$

where $\dot{\theta}_i^-$ is the angle of deflection of the $i$th string just before the crawler crosses over the $i$th particle and $\dot{\theta}_i^+$ is the angle of deflection right after the transit.

## III. Numerical Studies

The equations of motion of the flexible space tether system with the crawler mass, Eqs. (12) and (13), are simulated numerically employing the physical parameters of the project MAST (multi-application survivable tether experiment) planned by Tether Unlimited, Inc. (TUI) [11]. The orbit is assumed to be circular with altitude 400 km and the coordinate system is defined with the $x$ axis in the flight direction and the $z$ axis in the direction toward the center of the Earth. The tether with length 1000 m is modeled as 101 particles including the mother satellite and subsatellite as the end bodies. Each length is then 10 m for the massless strings connecting the particles. Table 1 summarizes the parameters employed in the simulations.

The initial condition is selected so that the tether system is in an equilibrium state hanging down along the $z$ axis with the mother satellite at the top and the subsatellite at the bottom of the tether. The crawler mass is placed on the tether system at a position displaced from the center of mass of the total system, in which the crawler mass will maintain its position without any force. The equilibrium position is approximately 620 m distant from the mother satellite. The crawler mass at the off equilibrium position starts to move in the direction of the greater force between the gravitational force and the centrifugal force in the orbit.

The result of numerical simulation is shown in Figs. 2–6. In the first case, the initial displacement $S_0$ of the crawler mass is set to be 600 m downward from the mother satellite, $m_0$, along the tether length ($S_0 = 600$ m), which is about 25 m higher than the position of the center of mass of the total system. Figure 2 shows stroboscopic pictures of the tether system in the orbital plane with time increasing from 0 s at the initial condition to 4816 s. The origin of the figure is on the orbit and the amplitude of motion in the direction of the $x$ axis is shown in an enlarged manner in comparison with that in the direction of the $z$ axis to show the motion clearly. The crawler mass is shown as a point on the tether system placed a bit above the origin, that is, the center of mass of the system, and it is seen to move along the tether toward the position of the mother satellite. The crawler mass is being accelerated in the direction of the positive $z$ axis, or in other words, toward the zenith in a manner similar to a free elevation, and is seen to drift toward the positive direction of the $x$ axes due to the Coriolis force, which dominates as velocity increases. This results in movement of the mother satellite and the subsatellite in the negative $x$ direction and in the reverse direction through interacting movement of the total system in the early stage of the time period. It is seen that the crawler mass joins with the mother satellite at the final stage. Figure 3 shows the trajectories of the mother $m_0$, the subsatellite $m_{N-1}$, and the crawler mass $m_N$, in the orbital plane for the case shown in Fig. 2, $S_0 = 600$ m.

Figure 4 shows the result of the case when the initial displacement of the crawler mass is set to 650 m downward from the mother satellite, which is about 25 m lower than the position of the center of mass of the total system ($S_0 = 650$ m). The crawler mass is seen to move, being accelerated in the direction of the negative $z$ axis, that is, toward the center of the Earth in a manner similar to a free fall because the crawler has been in a slightly lower orbit than the total mass center. Figure 5 shows the trajectories of the mother $m_0$, the subsatellite $m_{N-1}$, and the crawler mass $m_N$, in the orbital plane for the case shown in Fig. 4, $S_0 = 650$ m.

The resulting motions obtained in Figs. 2 and 4 are in the opposite directions which might be considered a natural feature in the present analysis. It should be noted that these two cases are not strictly symmetric in the dynamics sense, and as a result the computation time is different for these two cases.

It is quite interesting to compare the present result of the 101-mass model with that of a simplified analysis using six masses, that is, modeling the tether with only four particles, plus the mother satellite, and the subsatellite. Figure 6 shows the result of the trajectories of the

**Table 1 Parameters of the multi-application survivable tether experiment (gravity-gradient stabilized tether)**

| | |
|---|---|
| Circular orbit altitude: | 400 km |
| Tether length: | 1000 m |
| Line density: | 0.0004 kg/m |
| Mother satellite mass: | $m_0 = 0.6$ kg |
| Subsatellite mass: | $m_{N-1} = 1$ kg |
| Mass of crawler: | $m_N = 1$ kg |
| Initial condition (satellite): | Crawler mass is nominally $s_0$ (m) distant from the mother $m_0$, along the tether length. |

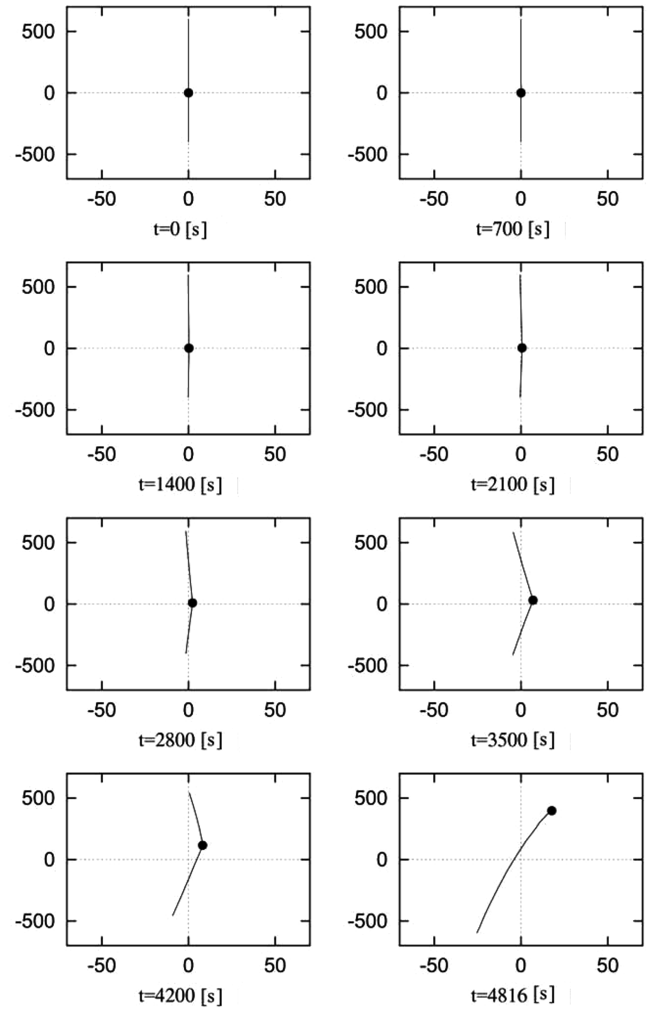


**Fig. 2 Stroboscopic figures of the system in the orbital plane ($S_0 = 600$ m).**

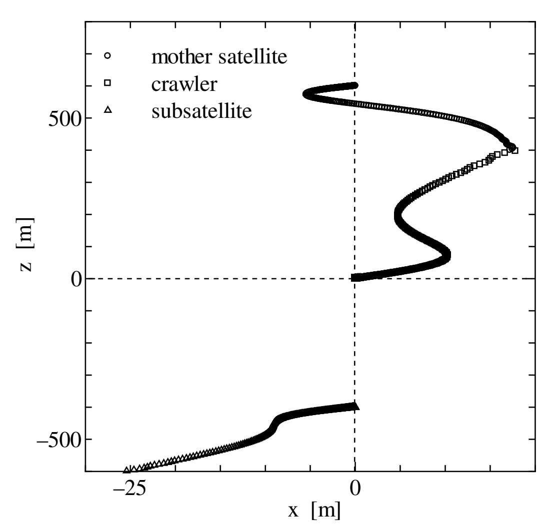


**Fig. 3 Trajectories of the crawler, the mother satellite, and the subsatellite in the orbital plane ($S_0 = 600$ m).**

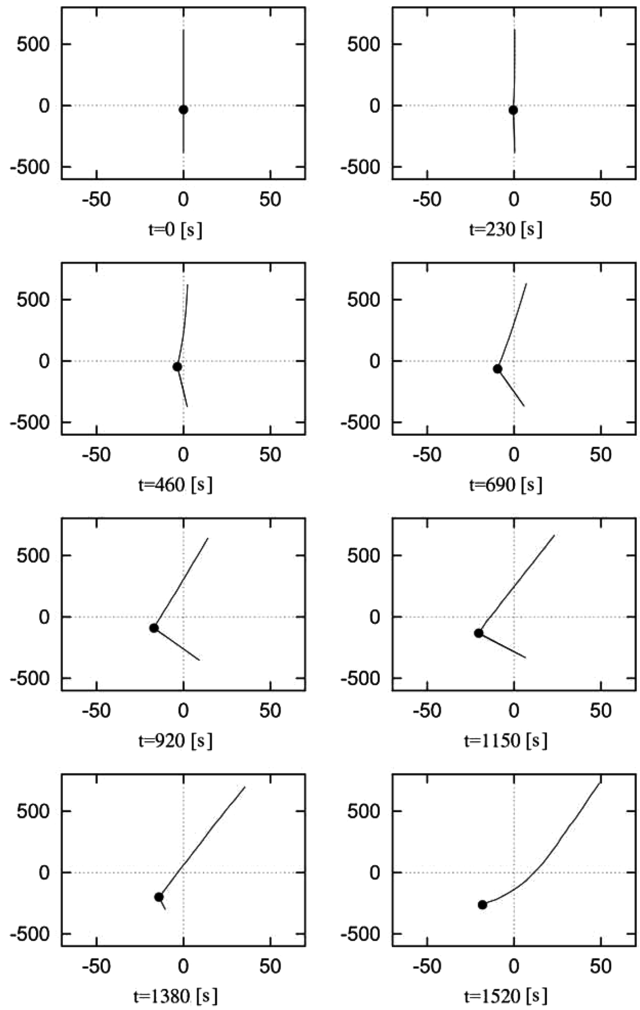


**Fig. 4 Stroboscopic figures of the system in the orbital plane ($S_0 = 650$).**

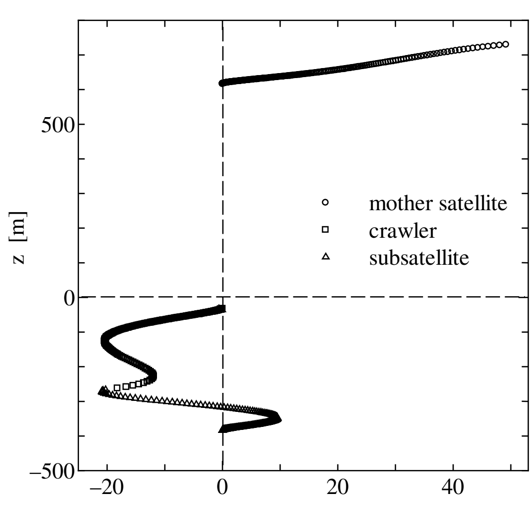


**Fig. 5 Trajectories of the crawler, the mother satellite, and the subsatellite in the orbital plane ($S_0 = 650$ m).**

mother satellite, the subsatellite, and the crawler mass in the orbital plane for the six-mass model ($S_0 = 600$ m). A significant difference is observed by comparing this result with that of the 101-mass model in Fig. 3 for the same initial conditions. One might expect this after imagining the difference in swaying motions between a chain rope with five elements and 100 elements with light counterweights at both ends.

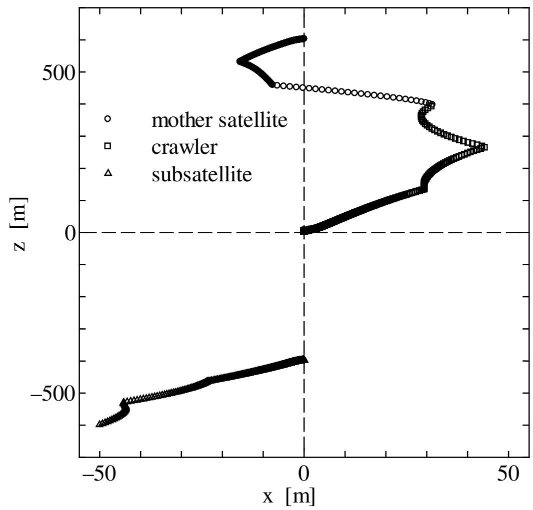


**Fig. 6 Trajectories of the crawler, the mother satellite, and the subsatellite in the orbital plane for the six-mass model ($S_0 = 600$ m).**

## IV. Conclusions

A method to formulate the equations of motion is introduced in this Note for a flexible tether system equipped with a crawler mass moving on the tether. The equations of motion are formulated by using Kane's method and a Lagrange multiplier method to satisfy the constraints. The motion of a crawler mass on the tether system is studied numerically resulting in rather complicated motions between the crawler mass and the flexible tether system.

Results of the numerical simulation studies indicate that the present formulation for studying crawler mass motion on a flexible tether is efficient and leads to reasonable results. Comparison with a simplified analysis assures that the present analysis is able to capture important effects that are not properly predicted by the simplified approaches.

## References

[1] Beletsky, V. V., *Dynamics of Space Tether Systems*, American Astronautical Society, San Diego, CA, 1993, p. 20.

[2] Cosmo, M. L., and Lorenzini, E. C., *Tethers In Space Handbook*, 3rd ed., NASA MSFC, Dec. 1997.

[3] Fujii, H. A., "New Space Technology: 1 km Tether to 100,000 km Space Elevator," *The International IPSI-2004 MONTENEGRO Conference*, Opening Keynote Speech, Sveti Stefan, Montenegro, IPSI, Belgrade, 2–9 Oct. 2004.

[4] Hoyt, R. P., and Forward, R. L., "Failsafe Multi-Strand Tether SEDS Technology," *4th International Conference on Tethers in Space*, American Astronautical Society, Washington, D.C., April 1995.

[5] Fujii, H. A., Takegahara, H., Oyama, K., Sasaki, S., Yamagiwa, M., Krujff, M., Van der Heide, E. J., Sanmartin, J. R., and Charro, M., "A Proposed Bare-Tether Experiment on Board a Sounding Rocket," *AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Reston, VA, 16–19 Aug. 2004.

[6] Bradley, C. E., and Shelef, B., "Results from the First Annual Space Elevator Climber Competition," IAC Paper 05-D4.3.01, 20 Nov. 2005.

[7] Pascal, M., Djebli, A., and El Bakkali, L., "A New Deployment/Retrieval Approach Between Crawler and Conventional Systems," *Journal of Applied Mathematics and Mechanics*, Vol. 65, No. 4, 2001, pp. 689–696.
doi:10.1016/S0021-8928(01)00073-9

[8] McInnes, C. R., "Dynamics of a Particle Moving Along an Orbital Tower," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, March–April 2005, pp. 380–382.

[9] Lorenzini, E. C., "A Three-Mass Tethered System for Micro-g/Variable-g Applications," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 3, May–June 1987, pp. 242–249.

[10] Tohyama, S., Watanabe, N., and Tano, H., "Effective Method to Analyze Mechanism with Closed Loop: Part 1 Analytical Method in Recurrence Formula," *Journal of Japanese Robot Association*, Vol. 14, No. 2, 1996, pp. 279–286 (in Japanese).

[11] Hoyt, R., and Slostad, J., "The Multi-Application Survivable Tether (MAST) Experiment," AIAA Paper 2003-5219, 20–23 July 2003.